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Relativistic many-body theory with radioactive ion beams: Surface pion condensation

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Abstract. We critically review the present relativistic mean-field theory from the viewpoint of missing pions. We introduce the interesting experimental data on pionic states taken at RCNP. These data seem to suggest the occurrence of pion condensation in the nuclear surface. Qualitative discussion is made on the consequence of surface pion condensation on Gamow-Teller transitions and spin response functions and others. The radioactive ion beams are the tools of studying the unstable nuclei, which have extended nuclear surfaces. We shall start with radioactive ion beams the nuclear surface science, which includes the surface pion condensation as the important ingredient in addition to spin-orbit splitting and surface pairing.

PACS. 21.10.-k Properties of nuclei; nuclear energy levels – 21.10.Hw Spin, parity, and isobaric spin – 21.30.Fe Forces in hadronic systems and effective interactions – 21.60.Jz Hartree-Fock and random-phase approximations

1 Relativistic mean-field theory

I am very happy to be here to join the celebration of the 60th birthday of Prof. M. Ishihara. I would like to agree with others who stressed that Prof. Ishihara did many contributions to the Nuclear Physics community like developing a new field, building new facilities and promoting young scientists. I would like to add one more as he even gave strong motivation to theorists, whenever he met with us, by asking questions like what is new, is there something interesting and so on. This motivates theorists to think hard on something new. How it works, I would like to do in front of you now. I would start with my standard conclusion on relativistic theory first. The relativistic mean-field (RMF) theory is quantitatively very good. This is mainly because RMF includes the strong three-body repulsion, which is responsible for saturation, and strong spin-orbit force, responsible for magic numbers. However, this theory does not include pions, which should be the most important ingredient in hadron physics. Where are pions in nuclear ground states?

In fact, the Lagrangian of the relativistic mean-field (RMF) theory is

$$\begin{aligned}\mathcal{L} = \bar{\psi}\Big[& \imath\gamma^\mu\partial_\mu - M - g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu \\ & -g_\rho\gamma_\mu\tau^a\rho^{\mu a} - e\gamma_\mu\frac{(1-\tau_3)}{2}A^\mu\Big]\psi \\ & +\frac{1}{2}\,\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\ & -\frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3(\omega_\mu\omega^\mu)^2 \\ & -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ ,\end{aligned} \tag{1}$$

where the field tensors H, G and F for the vector fields are defined through

$$\begin{aligned}H_{\mu\nu} &= \partial_\mu\omega_\nu - \partial_\nu\omega_\mu\ , \\ G^a_{\mu\nu} &= \partial_\mu\rho^a_\nu - \partial_\nu\rho^a_\mu - g_\rho\,\epsilon^{abc}\rho^b_\mu\rho^c_\nu\ , \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu\ ,\end{aligned} \tag{2}$$

and other symbols have their usual meaning. Here, σ denotes the scalar meson, ω the vector meson and ρ the isovector-vector meson. A denotes the photon.

This Lagrangian apparently does not include the pion. The pion term should be there, but this term is neglected in the RMF approximation due to the conservation of parity and isospin. There are six parameters in the RMF Lagrangian. Under the mean-field approximation we can construct coupled differential equations for nucleons and mesons with photon field, which could be easily solved numerically. By adjusting the parameters to the existing data on binding energies and radii of proton magic nuclei, these parameters are fixed by Sugahara and Toki [1]. We find good descriptions of binding energies and radii as shown in fig. 1. We mention here though that we have to take either two parameter sets (TM1 for $A \geq 40$ and TM2 for $A \leq 40$) or TMA with smooth mass dependence in order to describe nuclei in the entire mass regions. The

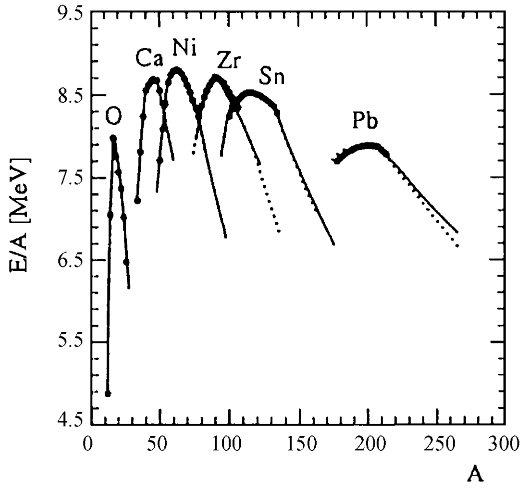

Fig. 1. The binding energy per particle as a function of the mass number for proton magic number nuclei. The experimental data are shown by dots and the TMA results are shown by solid curves and the NL1 results by dashed curves. Taken from ref. [1].

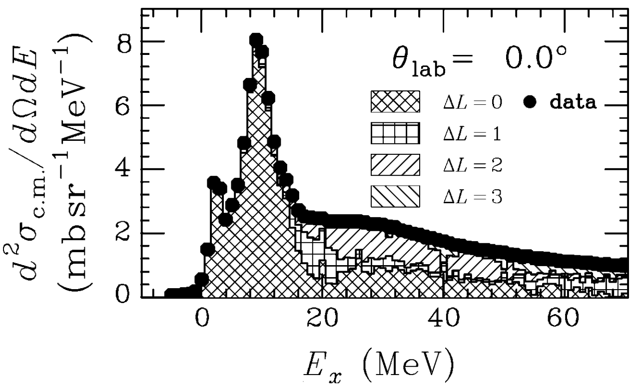

Fig. 2. The zero-degree (p,n) spectrum on ^{90}Zr decomposed into various multipole components. The GT strength is indicated by the hatched area. Taken from ref. [3].

goodness of the parameter sets has been demonstrated by calculating all the even-even mass nuclei in the entire mass region [2]. In this figure, the NL1 results are obtained with the use of the non-linear parameter set NL1 of the Frankfurt group [1].

In addition, we have calculated giant resonances, equation of state of nuclear matter and superheavy elements. We are very much satisfied with the performance of the RMF theory with the TM parameter sets. If we look, however, at the hadron physics, chiral symmetry and its spontaneous breaking are the essential ingredients for the successful description of the experimental data. Monte Carlo calculations of light nuclei suggest that large fraction of the binding energy is due to the pion. Why then does the RMF theory without pion work so well? Is there any site where the pion is necessary? The interesting place to look for the answers to these questions is the pionic excitation of nuclei.

2 RCNP (p,n) spin experiments

There are two important experiments on pionic excitations performed at RCNP using the (p,n) reactions by Sakai group [3,4]. They are the zero-degree spectra in the (p,n) reactions and the large momentum transfer (p,n) reactions at $E \sim 300$–$350\,\mathrm{MeV}$.

We start with the zero-degree (p,n) reactions. We take ^{90}Zr as an example [3]. Taking a simple shell model, we expect two states to be populated by the Gamow-Teller operator: the $g_{7/2}g_{9/2}^{-1}$ particle-hole state and the $g_{9/2}g_{9/2}^{-1}$ state. Due to a repulsive interaction between two states, we expect that the two states mix and that the higher state carries most of the strength, which is called the giant Gamow-Teller (GT) state. The experimental data are obtained at forward angles with an incident energy of 350 MeV and are analyzed as multipole components. The spectrum is shown in fig. 2 [3]. The large fraction of the GT strength is then found shifted further up to states higher than the giant GT state by 30 to 40 percent. This shift of the GT strength to higher states was studied theoretically as a coupling of the GT state to 2p-2h states due to the strong tensor force. With the coupling of highly excited states, the perturbative calculations are able to provide large strengths in the continuum. However, the change of the GT strengths seems too great, if one considers that the calculations are done perturbatively.

The GT strength seems to be exhausted by nucleon degrees of freedom. This fact suggests that the coupling of the GT states to delta isobars is very small. In terms of g' for delta-hole coupling in the spin-isospin channel, we have $g'_\Delta \leq 0.3$. This fact indicates that the delta-hole states should contribute largely to pion condensation and its precritical phenomenon. This possibility has to be confronted with hitherto missing observations of precritical phenomena [5,6]. The relativity is suggested to reduce the pionic collectivity, but it seems not enough [7]. The precritical phenomenon is expected also in the spin response functions [8]. Hence,we discuss now the second experiment of the Sakai group.

It is the (p,n) reaction with the measurement of polarizations in the initial and final channels at large momentum transfer. In addition, the isovector transfer is identified as 1 by the use of the (p,n) reaction. This spin measurement is able to provide the response functions in the pion and the ρ-meson channels [4].

In the standard model of spin correlations, we use the interactions in the pion (longitudinal) and ρ-meson (transverse) channels as

$$V_\pi = \frac{f^2}{m^2q^2}\left(g' - \frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m^2}\right)\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}\,\vec{\tau}_1\cdot\vec{\tau}_2\,, \tag{3}$$

$$V_\rho = \frac{f^2}{m^2q^2}\left(g' - C_\rho\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\rho^2}\right)\vec{\sigma}_1\times\vec{q}\cdot\vec{\sigma}_2\times\vec{q}\,\vec{\tau}_1\cdot\vec{\tau}_2\,. \tag{4}$$

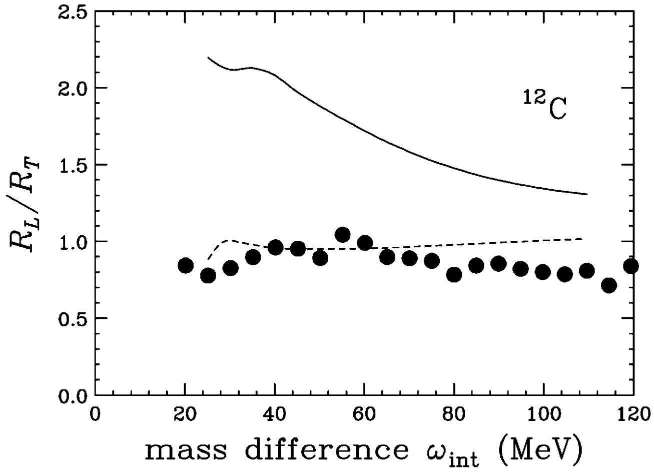

Fig. 3. The ratio of the longitudinal to transverse spin response functions in (p,n) reaction on ^{12}C as a function of the excitation energy. The experimental data are shown by dots. The DWBA calculation with the pion and ρ-meson correlations is shown by the solid curve, while the one without the correlations is shown by the dashed curve. Taken from ref. [4].

These interactions provide strong attraction in the pion channel and strong repulsion in the ρ-meson channel. Naturally then we expect that the pion response is softened from the non-interacting case and the ρ-meson response is hardened. This tells that the ratios of the spin longitudinal (π) and the spin transverse (ρ) responses are larger than 1 at smaller excitation energy.

The experimental data show surprisingly that the ratios are not larger than one. This is shown in fig. 3 for the case of ^{12}C [4]. This fact is difficult to understand in the present framework, when the response functions are due to a one-step process in the reaction mechanism. The relativistic description of the response functions is not enough to turn around the ratios, unless we use different g''s for the pion and the ρ-meson channels [9]. These two experimental data indicate that there is a serious problem in the present theoretical framework in the pion channel (spin-isospin channel).

3 Surface pion condensation

We propose to take the pion terms seriously now, which should be present in the Lagrangian. For simplicity of writing, we write explicitly only the σ and pion terms in the Lagrangian density as

$$\mathcal{L} = \bar{\psi}[\imath\gamma^\mu\partial_\mu - M - g_\sigma\sigma - g_\pi\gamma_5\gamma^\mu\partial_\mu\tau_a\pi^a]\psi + \mathcal{L}_{\rm meson}. \quad (5)$$

We then assume that the expectation value of the pion field is finite.

We write the equation of motion for nucleons and pions as

$$[\imath\gamma^\mu\partial_\mu - M - g_\sigma\sigma - g_\pi\vec{\nabla}\pi^a\gamma_5\vec{\gamma}\tau_a]\psi = 0 \quad (6)$$

and

$$[\vec{\nabla}^2 - m_\pi^2]\pi^a = -g_\pi\vec{\nabla}\langle\bar{\psi}\gamma_5\vec{\gamma}\tau_a\psi\rangle. \quad (7)$$

The σ-meson and others follow the same equations of motion as the standard case. These equations tell the reason why we have not included the pion mean field until now. The source term of the pion Klein-Gordon equation is non-vanishing only when the parity and the isospin are mixed in the single-particle state. This violation of the parity and isospin is caused by the pion term in the above Dirac equation for nucleons. Hence, the single-particle state can be expressed as

$$\psi_{n,jm}(x) = \sum_{\kappa,t} W^n_{\kappa,t}\phi_{\kappa jm,t}\,. \quad (8)$$

Here, $\phi_{\kappa jm,t}$ denote the eigenfunctions of nucleons without the pion mean-field term. The summation over κ means the parity mixing and the summation over t the isospin mixing. We would call the case of non-vanishing pion mean field as surface pion condensation, since the pion source term involves derivative of the mean field. This is a very important statement, since pion condensation has been considered to occur only at high densities like two to three times the saturation density. This large critical density is caused by the necessity of making the nuclear matter non-uniform in order to make the derivative in (7) active. On the other hand, in finite nuclei the matter density changes naturally and the strong pion correlations can utilize this density variation. In fact, the recent numerical calculations of Sugimoto *et al.* demonstrate the finite pion mean field [10].

We discuss here the qualitative consequence of surface pion condensation. First, we discuss the Gamow-Teller transitions. Without pion condensation, there exist only two transitions as discussed above for ^{90}Zr. However, the mixing of parity and isospin allow transitions of many-neutron states to many-proton states. This makes the spectrum of the Gamow-Teller transitions with some GT strengths above the two dominant peaks. Hence, without the strong mixing of two-particle two-hole states due to the large tensor force, we can have large strengths in the continuum in the simple mean-field theory as the experiment demands.

The longitudinal spin response functions are largely modified due to surface pion condensation. The response is caused by pionic correlations. Since large pionic correlations are used to construct the nuclear ground state, the pionic fluctuation ought to be reduced largely. This effect should make the spin response in the pion channel very weak. We have to really work out the spin response functions both in the longitudinal and in the transverse channels in the surface pion condensed case and compare quantitatively with the experimental data [4].

The surface pion condensation provides us the possibility to describe light-mass nuclei using the same parameter set as the one for heavy-mass nuclei. This is because the light-mass nuclei have larger proportion of surface area to volume as compared to heavy ones and the surface pion condensation is active at the nuclear surface.

There should be many other consequences of surface pion condensation in nuclear phenomena. Pairing correlations and spin-orbit couplings are all surface phenomena and surface pion condensation would couple with these correlations and provide various interesting phenomena.

4 Conclusion

We have discussed the possible occurrence of surface pion condensation in order to understand the recent (p,n) experimental data taken at RCNP. This suggestion is motivated by the missing pion contribution in the discussion of ground states of finite nuclei, while pions are essential for hadron physics. We have made qualitative discussions on the consequence of surface pion condensation to the Gamow-Teller strengths, the spin response functions and the ground-state binding energies.

In relation with the radioactive ion beams, RIB provide a tool to study unstable nuclei, which have larger surface areas due to the small binding of valence particles. This possibility opens a new exciting field related with the nuclear surface. Pairing correlations provide pair condensation, which mixes particle numbers. The spin-orbit interaction provides the spin-orbit splitting, which mixes the spin and orbit quantum numbers. Newly, the pionic correlations may lead to surface pion condensation, which mixes the parity and isospin. These are extremely interesting phenomena of strong interaction physics.

The author is grateful to Prof. K. Ikeda for fruitful discussions on surface pion condensation. In particular, he came to this possibility through completely different arguments. This discussion gives the author the confidence of working out the numerical calculations, which are being worked out with S. Sugimoto. The recent numerical calculations have confirmed surface pion condensation to occur in the RMF calculation. The author is also very grateful to Dr. Wakasa for letting him use his figures for presentation and for fruitful discussions.

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